How Simple Can It Be to Construct Good Codes

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Outline

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AWGN Channels

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- Input: $x_t \in \mathcal{A}$, where $\mathcal{A} \subset \mathbb{R}^\ell$ is a signal constellation of finite size.
- Output: $y_t = x_t + w_t$, where w_t is an ℓ-dimensional sample from a white Gaussian noise process with power spectrum density (PSD) σ^2.
- The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = \frac{\sum_{x \in \mathcal{A}} ||x||^2}{\ell \sigma^2 |\mathcal{A}|}$$

- The maximum transmission rate (bits/dimension) when the signal points are used with equal probability is given by $I(X; Y)$, which is naturally upper-bounded by $0.5 \log(1 + \mathrm{SNR})$.
- *Roughly speaking*, by a good code, we mean a code that performs *well* within one dB from the corresponding Shannon limit.

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- Non-binary, BICM, $\cdots$

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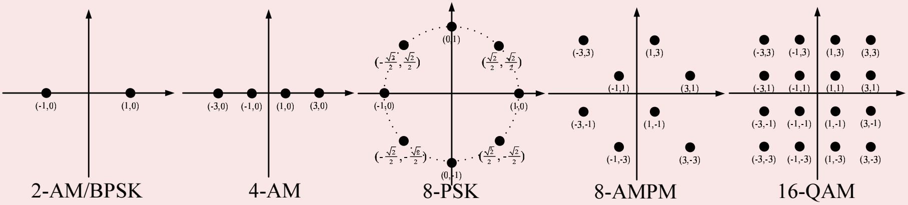

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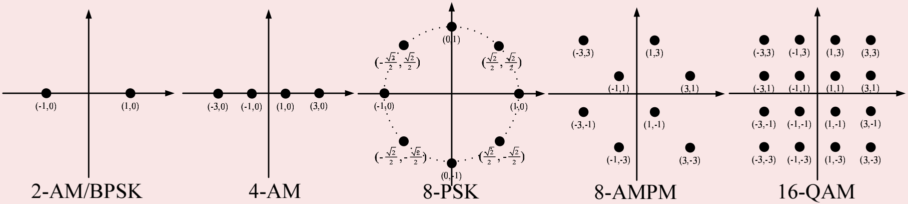

- any given target error performance (of interest), say, 10^{-4}, 10^{-6}, or 10^{-15}.

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Any Signal Constellation Can Be Treated As A Group

A finite *abelian group* $(\mathcal{A}, +)$ is a finite set $\mathcal{A}$ along with a binary operation $+ : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ satisfying the following rules.

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3. There is a unique element $\theta \in \mathcal{A}$ satisfying $\alpha + \theta = \alpha$ for all $\alpha \in \mathcal{A}$. We call θ the *identity element* of $\mathcal{A}$ and denote it by 0 for convenience.
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   set: $\mathcal{A} = \Lambda/\Lambda_0$, where $\Lambda \subset \mathbb{R}^\ell$ is a lattice with Λ_0 as a sub-lattice;
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3. General constellation:
   set: indexed (in an arbitrary order) by $\{0, 1, \cdots, q-1\}$;
   add: $\alpha + \beta \mod q$ for two signal points α, β.

RSPC Codes over Groups

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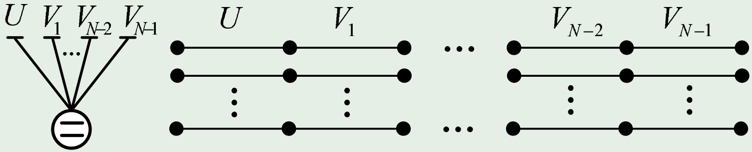
2. *Decoding:* The soft-in soft-out (SISO) decoding can be implemented in the same way as the message processing at a generic variable node of an LDPC code (binary or non-binary).



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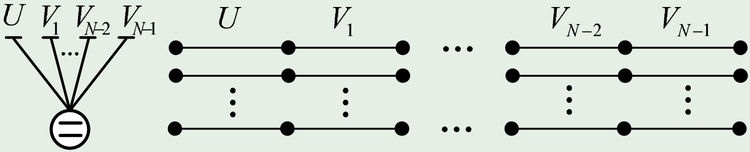

3. *Complexity:* No computational load is required for encoding; $O(|\mathcal{A}|)$ per coded symbol for decoding.

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4 *Performance:*

- For BPSK signalling over the AWGN channels, $\mathcal{A} = \{-1, +1\}$, the symbol-error-rate (SER) (i.e., the bit-error-rate (BER)) of the R code is given by

$$p_R(\mathrm{SNR}) = Q\left(\sqrt{\frac{N}{\sigma^2}}\right), \tag{1}$$

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[Zhuang13] Qiutao Zhuang, Jia Liu, and Xiao Ma, "Upper bounds on the ML decoding error probability of general codes over AWGN channels," [Online]. Available: `http://arxiv.org/abs/1308.3303`.

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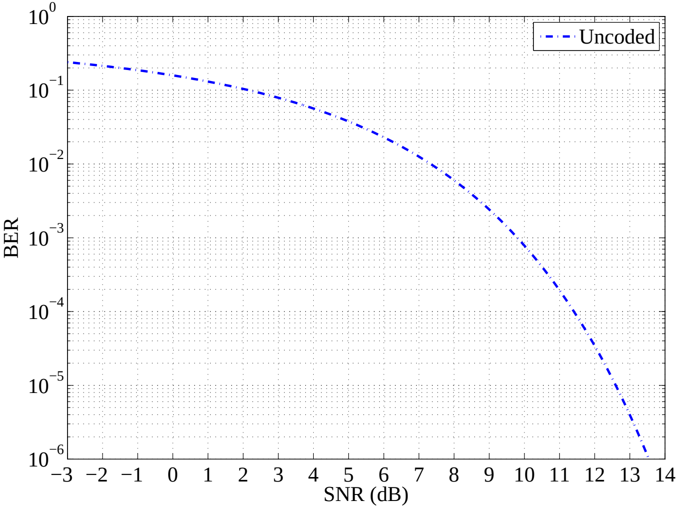

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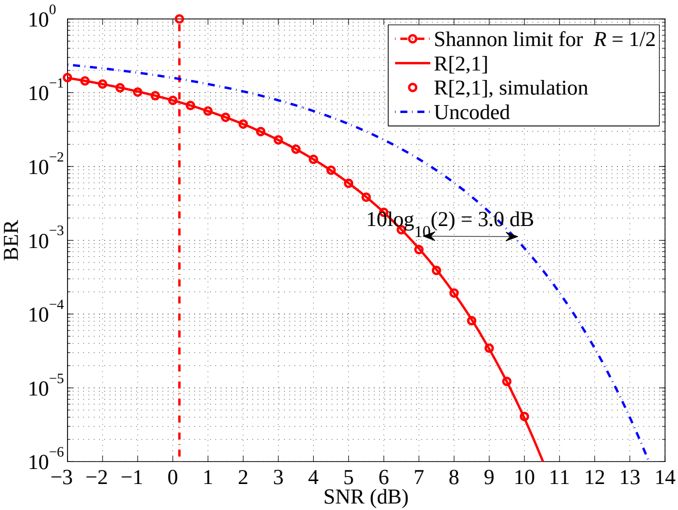

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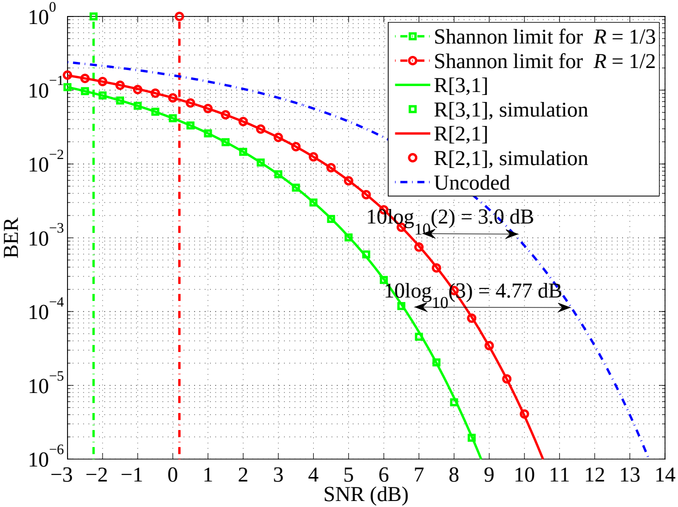

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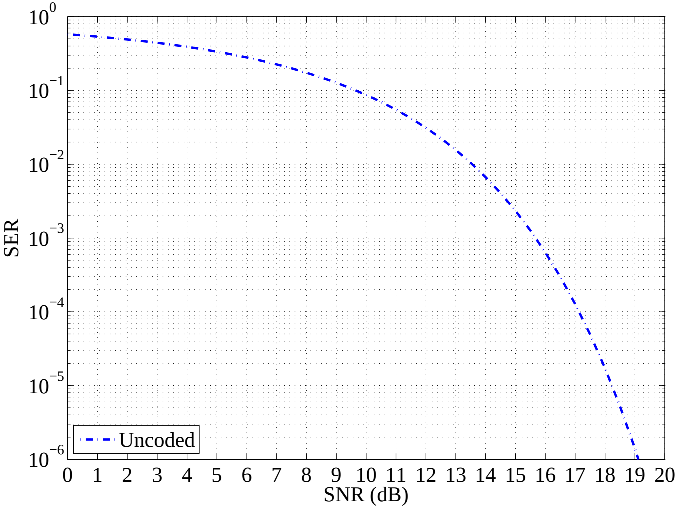

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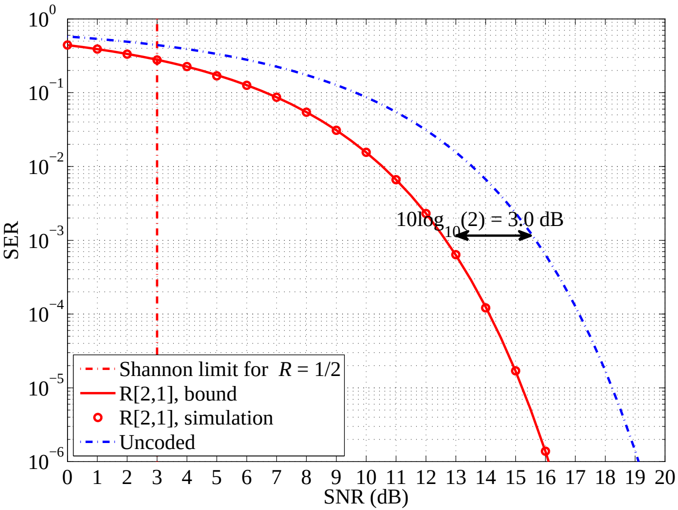

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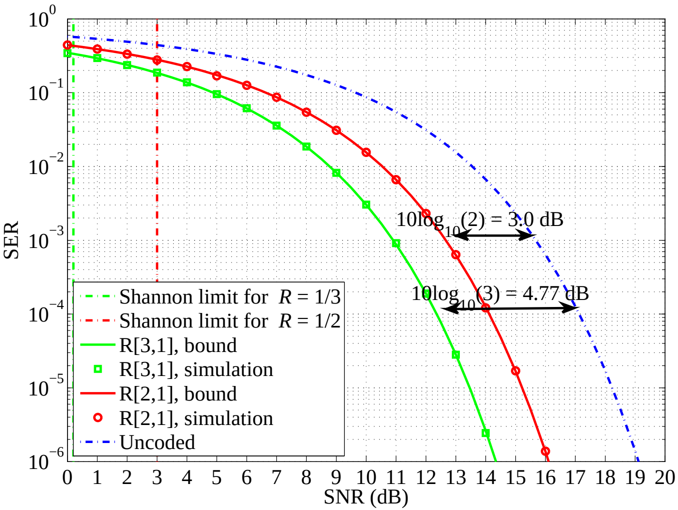

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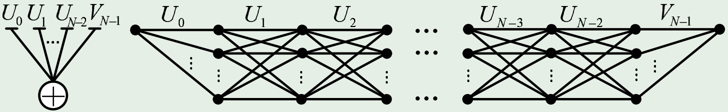

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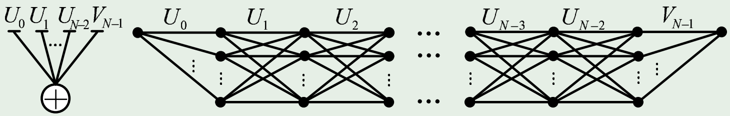

3. *Complexity:* $O(1)$ per coded symbol for encoding; $O(|\mathcal{A}|^2)$ per coded symbol for decoding.

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$$p_{SPC}(\text{SNR}) \leq \sum_{i=1}^{N-1} \frac{i}{N-1} \binom{N-1}{i} Q\left(\sqrt{\frac{\lfloor 2(i+1)/2 \rfloor}{\sigma^2}}\right), \quad (2)$$

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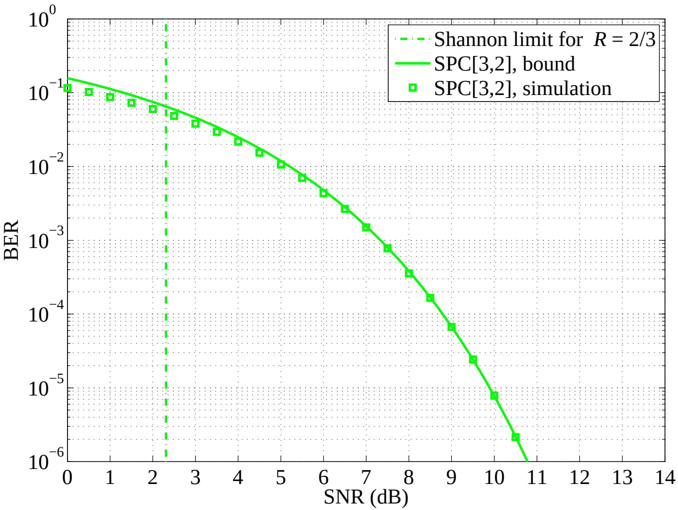

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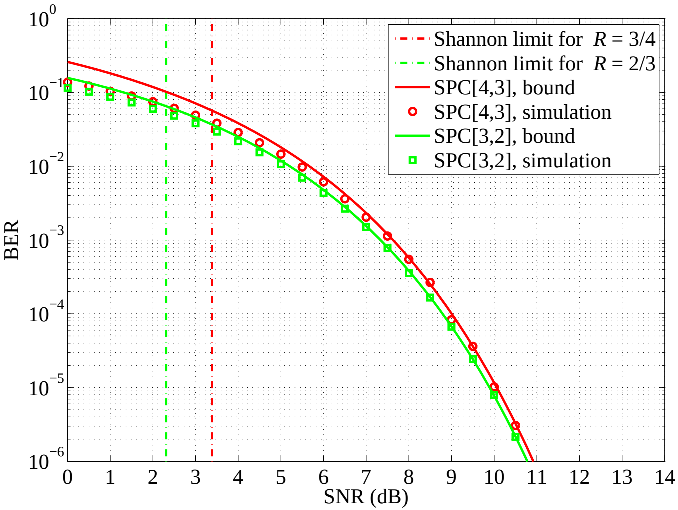

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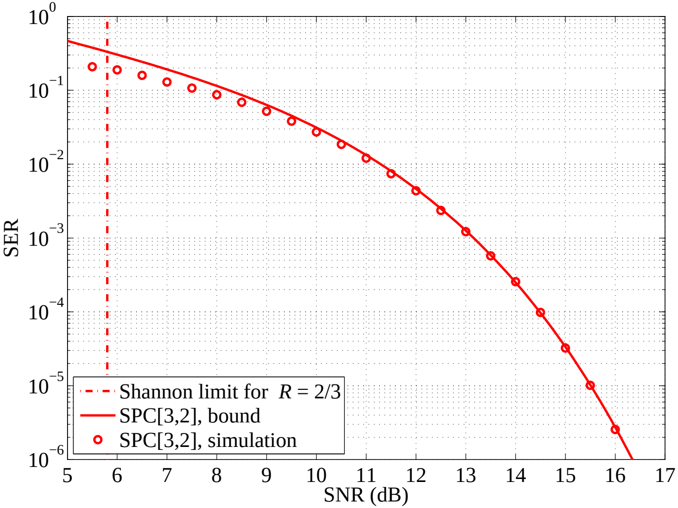

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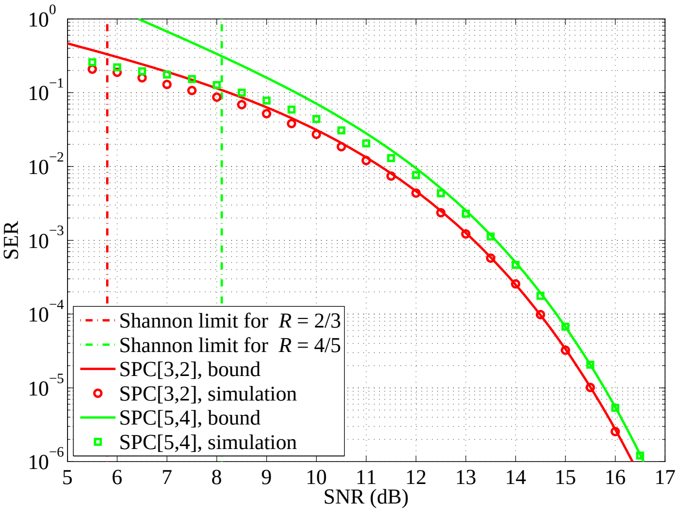

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Time-Sharing

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$$0 < \cdots < \frac{1}{N} < \frac{1}{N-1} < \cdots < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \cdots < \frac{M-2}{M-1} < \frac{M-1}{M} < \cdots < 1.$$

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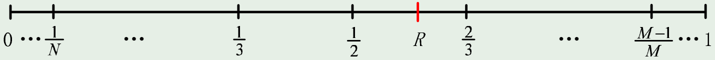

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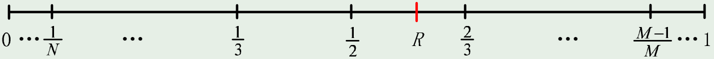

3 For any given code rate $R = \frac{P}{Q}$ (P and Q are co-prime), we can find a (unique) small interval $(\frac{K_L}{N_L}, \frac{K_U}{N_U})$ such that $\frac{K_L}{N_L} < \frac{P}{Q} \leq \frac{K_U}{N_U}$.

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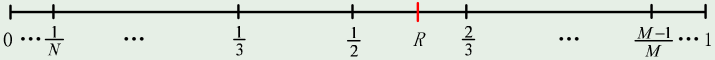

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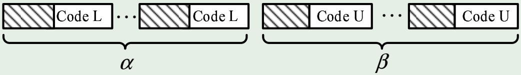
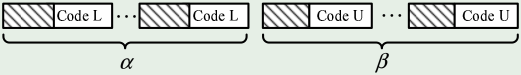
4. By time-sharing, i.e., using the code $\mathscr{C}[N_L, K_L]$ α times and the $\mathscr{C}[N_U, K_U]$ β times, we can construct a code with rate $R = \frac{P}{Q}$.



RSPC Codes over Groups

Time-Sharing

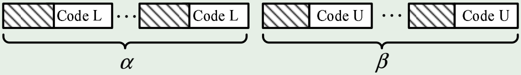


5 The time-sharing parameters α and β can be determined by

$$\frac{P}{Q} = \frac{\alpha K_L + \beta K_U}{\alpha N_L + \beta N_U}. \tag{3}$$

RSPC Codes over Groups

Time-Sharing



5 The time-sharing parameters α and β can be determined by

$$\frac{P}{Q} = \frac{\alpha K_L + \beta K_U}{\alpha N_L + \beta N_U}. \quad (3)$$

6 These codes are referred to as the RSPC codes. An RSPC code with rate $R = \frac{P}{Q}$ is denoted as $\mathscr{C}[Q, P]$ for convenience.

RSPC Codes over Groups

Time-Sharing

RSPC Codes over Groups

Time-Sharing

7 *Encoding:*

- The left-most αK_L symbols $\mapsto$ α codewords of $\mathscr{C}[N_L, K_L]$;
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RSPC Codes over Groups

Time-Sharing

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RSPC Codes over Groups

Time-Sharing

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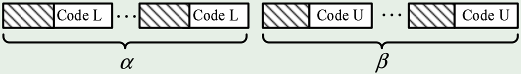
8 *Decoding:* The decoding is equivalent to decoding separately α codewords of $\mathscr{C}[N_L, K_L]$ and β codewords of $\mathscr{C}[N_U, K_U]$.

9 *Complexity:* $O(1)$ per coded symbol for encoding; $O(|\mathcal{A}|^2)$ per coded symbol for decoding.

RSPC Codes over Groups

Time-Sharing



10 *Performance:* The performance of the RSPC code with $R = \frac{P}{Q}$ is given by

$$p_{RSPC}(\text{SNR}) = \frac{\alpha K_L}{\alpha K_L + \beta K_U} \cdot p_L(\text{SNR}) + \frac{\beta K_U}{\alpha K_L + \beta K_U} \cdot p_U(\text{SNR}), \qquad (4)$$

where $p_L(\text{SNR})$ and $p_U(\text{SNR})$ are the performance functions for the code $\mathscr{C}[N_L, K_L]$ and the code $\mathscr{C}[N_U, K_U]$, respectively.

RSPC Codes over Groups

Example

Table: Examples of RSPC codes over groups

Groups	$R = \frac{P}{Q}$	$\left(\frac{K_L}{N_L}, \frac{K_U}{N_U}\right)$	α	β	Constructed codes
BPSK	$\frac{3}{8}$	$\left(\frac{1}{3}, \frac{1}{2}\right)$	2	1	$\mathscr{C}[3,1]^2 \times \mathscr{C}[2,1]$
BPSK	$\frac{5}{8}$	$\left(\frac{1}{2}, \frac{2}{3}\right)$	1	2	$\mathscr{C}[2,1] \times \mathscr{C}[3,2]^2$
8-PSK	$\frac{2}{5}$	$\left(\frac{1}{3}, \frac{1}{2}\right)$	1	1	$\mathscr{C}[3,1] \times \mathscr{C}[2,1]$
8-PSK	$\frac{3}{5}$	$\left(\frac{1}{2}, \frac{2}{3}\right)$	1	1	$\mathscr{C}[2,1] \times \mathscr{C}[3,2]$
16-QAM	$\frac{239}{255}$	$\left(\frac{14}{15}, \frac{15}{16}\right)$	1	15	$\mathscr{C}[15,14] \times \mathscr{C}[16,15]^{15}$

RSPC Codes over BPSK

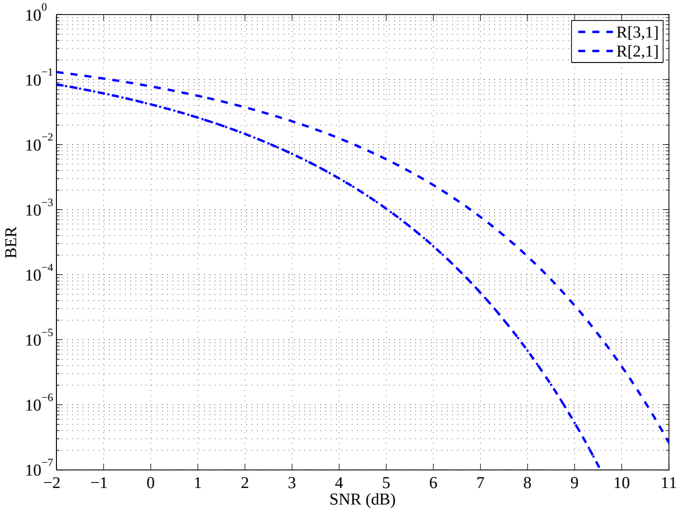
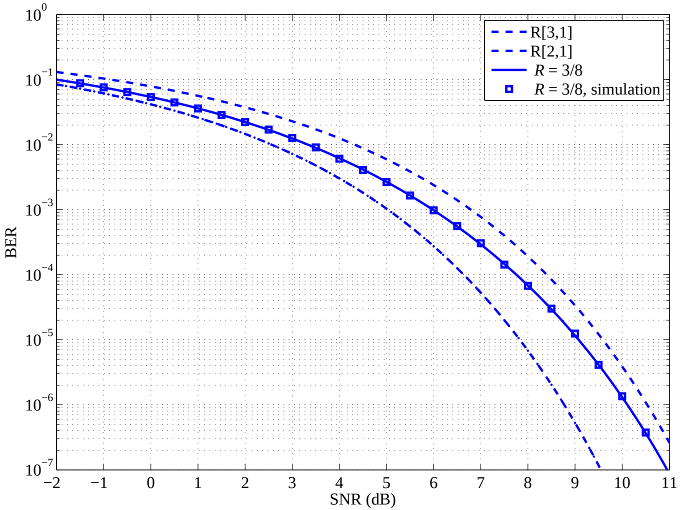


Figure: Performance of the RSPC codes $R = 3/8$ and $5/8$ over the BPSK-AWGN channels.

RSPC Codes over BPSK



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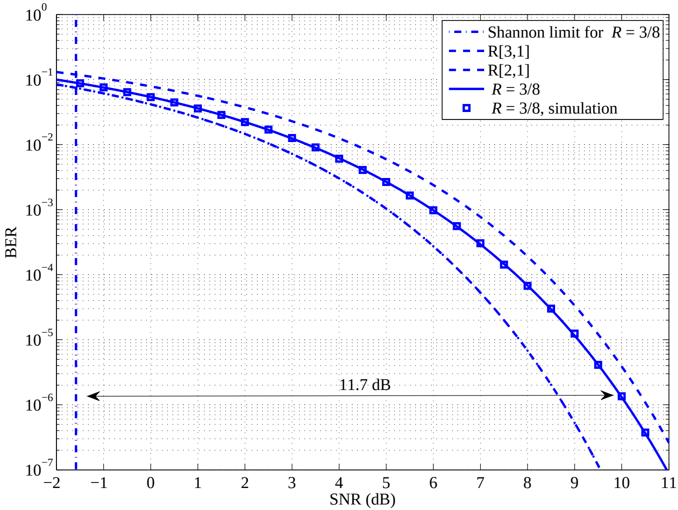

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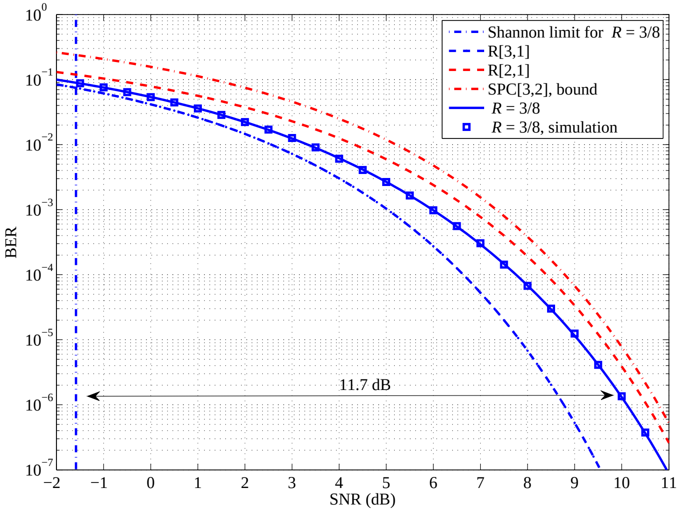

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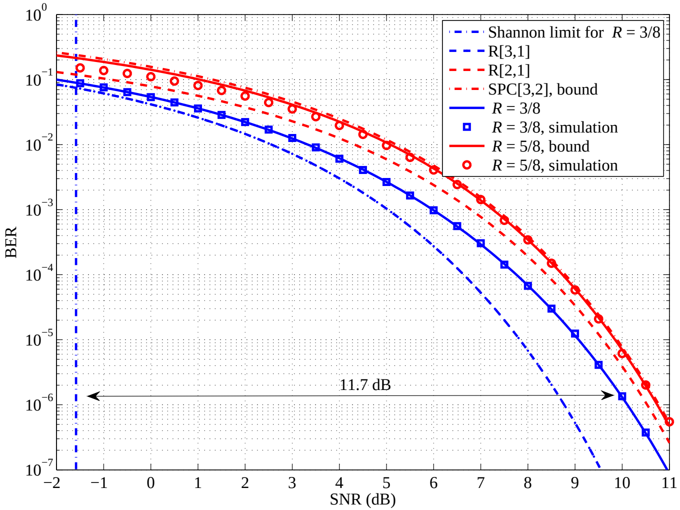

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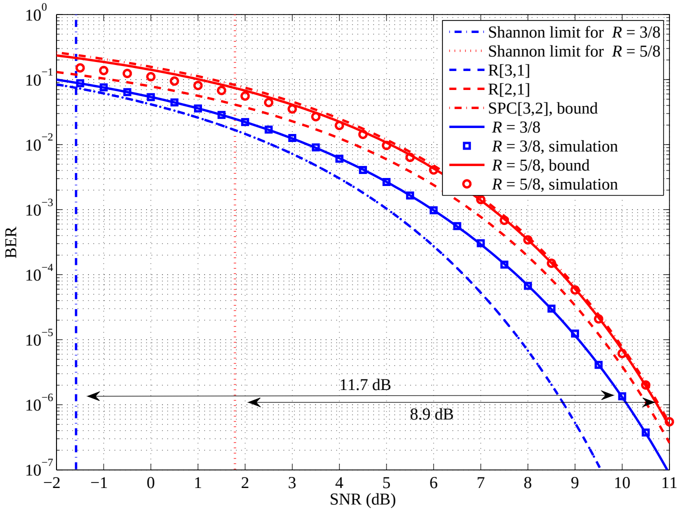

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RSPC Codes over 8-PSK

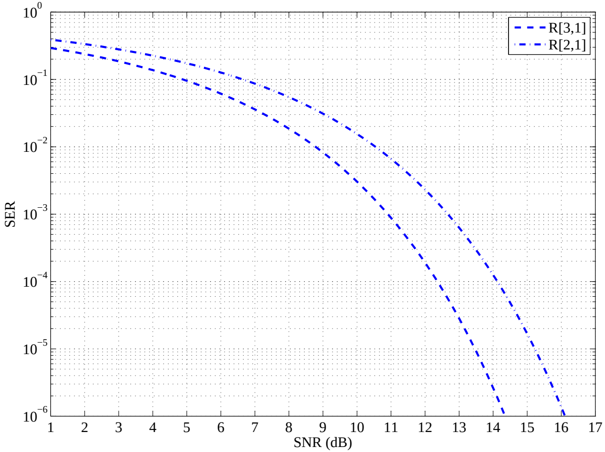
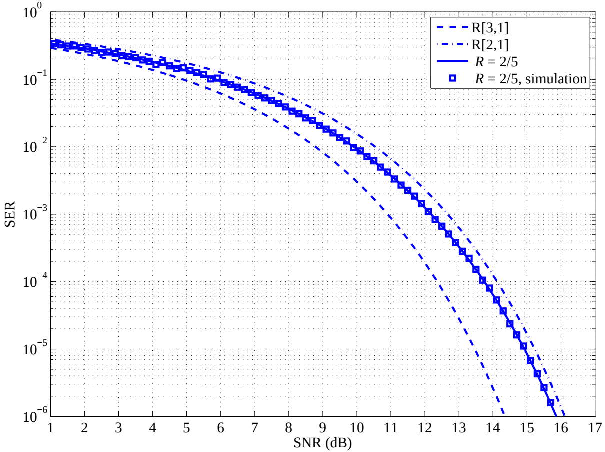


Figure: Performance of the RSPC codes with $R = 2/5$ and $3/5$ over the 8-PSK-AWGN channels.

RSPC Codes over 8-PSK



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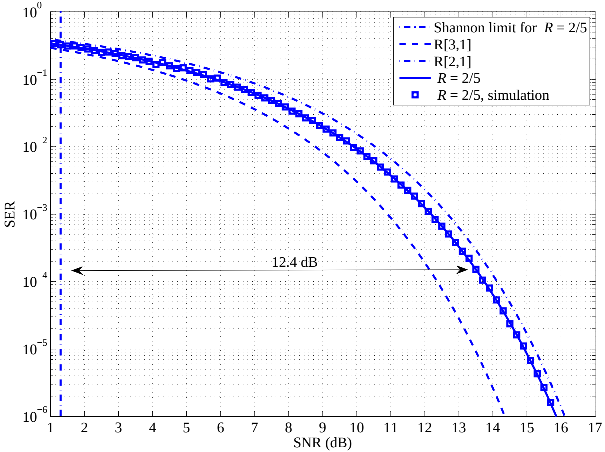

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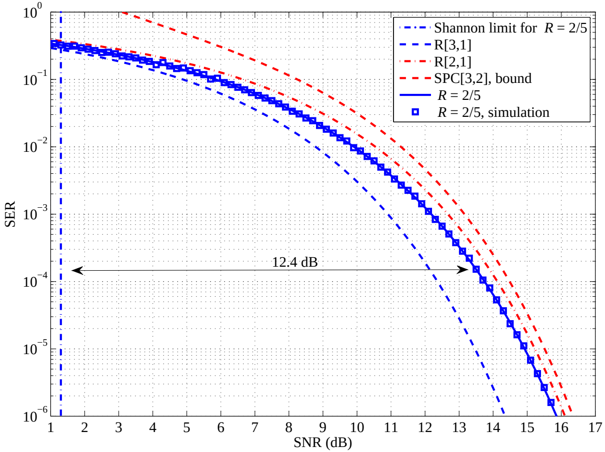

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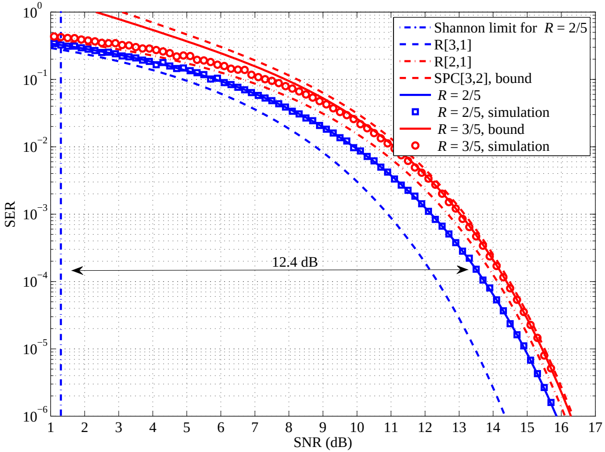

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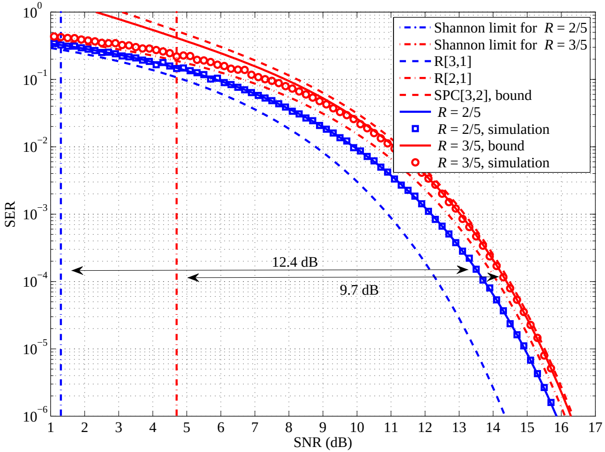

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RSPC Codes over Groups

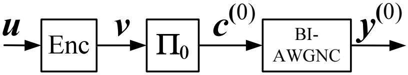
Summary

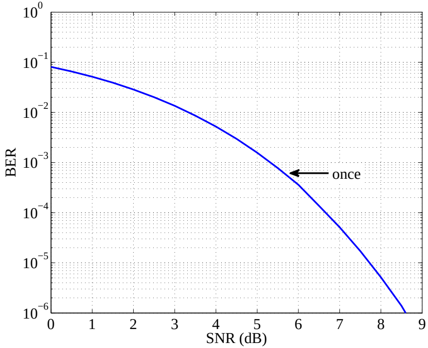
In summary, we are able to construct codes with

- any given rational code rate;
- analytic performance bounds;
- over any alphabet;
- but (usually) poor performance in terms of gap to the capacity.

Outline

Repetition Increases Reliability

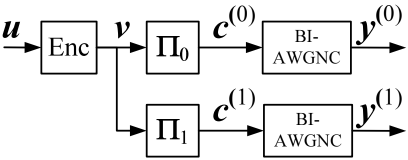


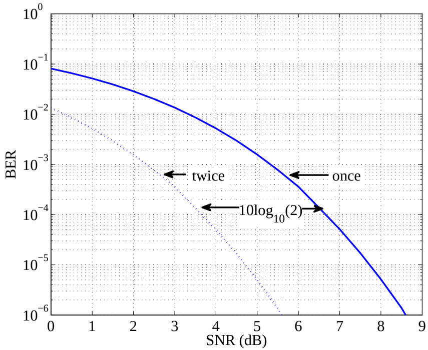


The codeword is transmitted once.

We assume a *basic code* $\mathscr{C}[n,k]$, whose performance curve in terms of BER versus SNR is available. In this talk, we assume that $\mathscr{C} = [N,K]^B$, the B-fold Cartesian product of a short block code $[N,K]$, consisting of all vectors of the form $(\boldsymbol{v}_0, \boldsymbol{v}_1, \cdots, \boldsymbol{v}_{B-1})$, where each $\boldsymbol{v}_i$ is a codeword in the $[N,K]$ code for $0 \le i \le B-1$.

Repetition Increases Reliability

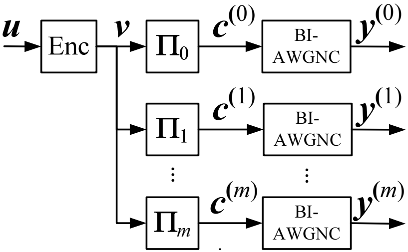


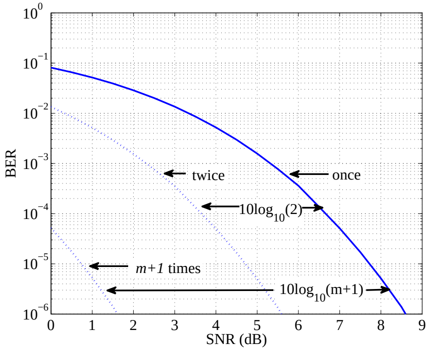


The same codeword is transmitted twice.

The performance curve shifts to the left by 3 dB.

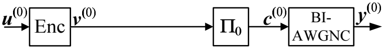
Repetition Increases Reliability

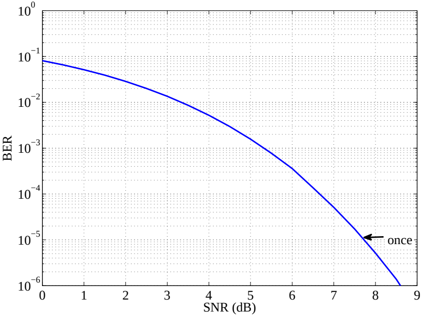




The same codeword is transmitted $m + 1$ times.

The performance curve shifts to the left by $10\log_{10}(m+1)$ dB.
Repetition increases reliability but decreases efficiency (code rate).

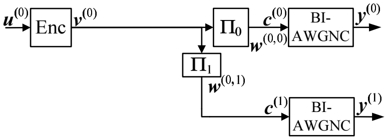


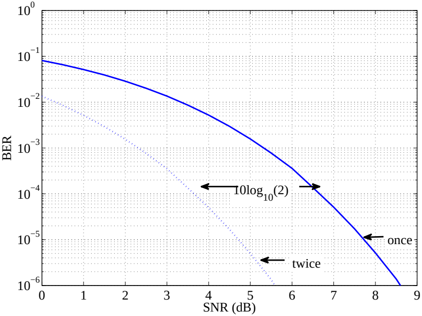


The first transmission

- Initially, the transmitter sends a codeword from the code $\mathscr{C}$ that corresponds to the first data block.

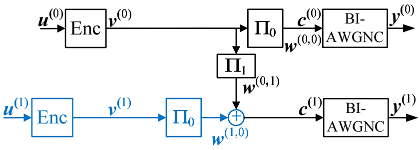
Superposition Increases Efficiency

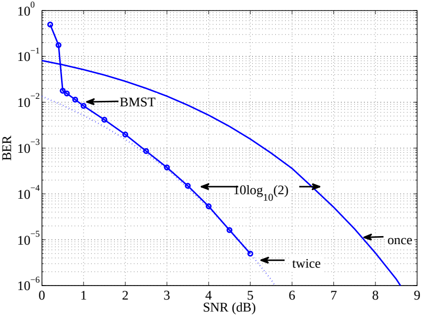




The second transmission

- Since the short code is weak, the receiver is unable to recover reliably the data from the current received block. Hence the transmitter transmits the codeword (possibly in its interleaved version) one more time.

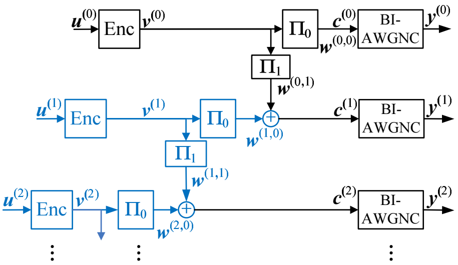


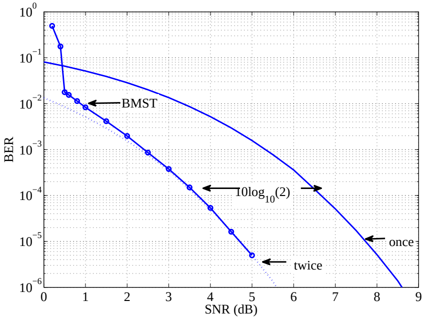


The second transmission

- Since the short code is weak, the receiver is unable to recover reliably the data from the current received block. Hence the transmitter transmits the codeword (possibly in its interleaved version) one more time.
- In the meanwhile, a fresh codeword from $\mathscr{C}$ that corresponds to the second data block is superimposed on the second block transmission.

Superposition Increases Efficiency

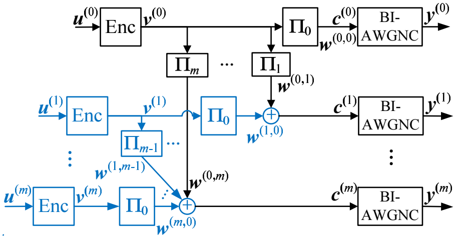


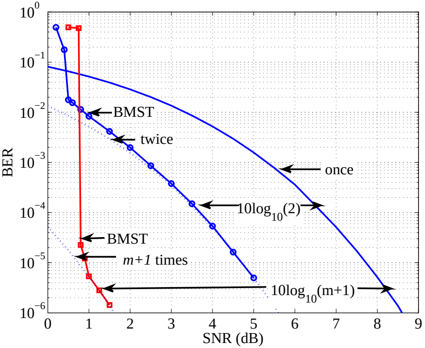


The third transmission for encoding memory $m = 1$

- In the third transmission, the current codeword $\boldsymbol{v}^{(2)}$ is superimposed to ("mixed into") the previous codeword $\boldsymbol{v}^{(1)}$ and then transmitted.
- This system can be iteratively decoded by passing extrinsic messages between adjacent layers. The performance is intuitively lower bounded by the repetition system.

Superposition Increases Efficiency



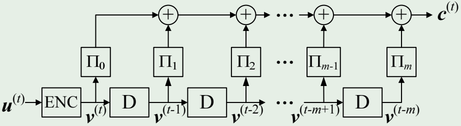


Transmission with memory m

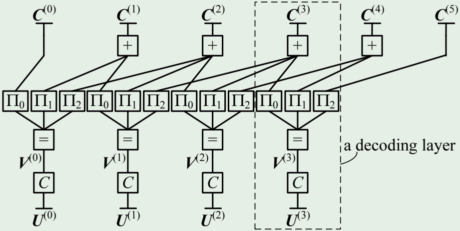
- Generally, for a BMST system with memory m, the t-th transmission is a superposition of the current codeword and the m consecutive past codewords, all in their randomly-interleaved version.
- The code rate remains *almost* the same, except that termination is needed, while the minimum distance increases very likely by m times for large $B \gg m$. Hence the error floor can be predicted by shifting the performance curve to the left by $10\log_{10}(m+1)$ dB.

The BMST system

- The encoding diagram of a BMST system with memory m.



- The normal graph for a BMST system with $L = 4$ and $m = 2$.



Summary

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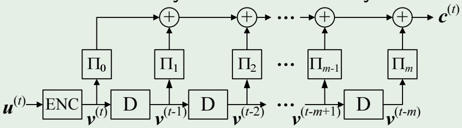

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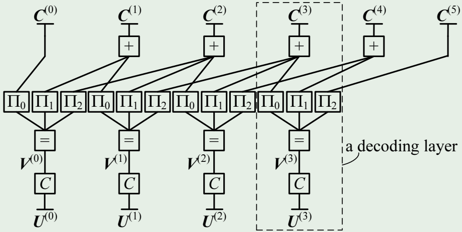

- The lower bound can be obtained by shifting the performance curve of the basic code to the left by $10\log_{10}(m+1)$ dB.

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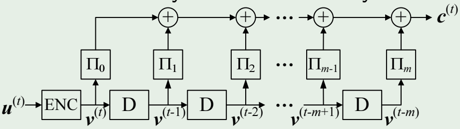

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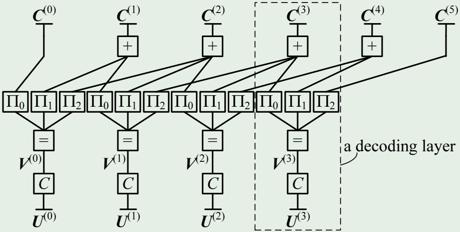

- The lower bound can be obtained by shifting the performance curve of the basic code to the left by $10\log_{10}(m+1)$ dB.
- Any code with fast encoding algorithm and SISO decoding algorithm can be embedded in the BMST system.

Outline

A General Procedure of Designing BMST-RSPC Codes over Groups

With the genie-aided lower bound, to construct a BMST-RSPC code of a given code rate $R = \frac{P}{Q}$ over a group $(\mathcal{A}, +)$ with a target SER p_{target}, we can perform the following steps.

1. Construct an RSPC code $\mathscr{C}[Q, P]$ over the group $(\mathcal{A}, +)$, whose performance (or upper-bound) function $\text{SER} = p_{RSPC}(\text{SNR})$ is available.

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3. Replace the encoder $\boxed{\text{ENC}}$ in the original BMST system by the RSPC encoder and $\boxed{+}$ by the addition over the associated group.

A General Procedure of Designing BMST-RSPC Codes over Groups

4 Find the required SNR to achieve the target SER. That is, find γ_{target} such that $p_{RSPC}(\gamma_{\text{target}}) \leq p_{\text{target}}$.

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$$m = \left\lfloor 10^{\frac{\gamma_{\text{target}}-\gamma_{\lim}}{10}} - 1 \right\rceil, \quad (5)$$

where $\lfloor x \rceil$ stands for the integer that is closest to x.

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where $\lfloor x \rceil$ stands for the integer that is closest to x.

7 Generate $m+1$ interleavers randomly.

Outline

Construction Examples – BMST-RSPC codes over BPSK

Table: The encoding memories required to approach the corresponding Shannon limits using BMST-RSPC codes at given target BERs

$R = P/Q$	Basic codes	p_{target}	γ_{target} (dB)	γ_{lim} (dB)	$\gamma_{\text{target}} - \gamma_{\text{lim}}$ (dB)	m
1/8	R $[8,1]^{1250}$	10^{-3}	0.77	−7.23	8.00	6
1/8	R $[8,1]^{1250}$	10^{-6}	4.51	−7.23	11.74	14
1/4	R $[4,1]^{2500}$	10^{-3}	3.78	−3.80	7.58	5
1/4	R $[4,1]^{2500}$	10^{-6}	7.52	−3.80	11.32	13
3/8	RSPC $[8,3]^{1250}$	10^{-3}	6.0	−1.6	7.6	5
3/8	RSPC $[8,3]^{1250}$	10^{-6}	10.1	−1.6	11.7	14
1/2	R $[2,1]^{5000}$	10^{-3}	6.79	0.19	6.60	4
1/2	R $[2,1]^{5000}$	10^{-6}	10.53	0.19	10.34	10
1/2	R $[2,1]^{5000}$	10^{-15}	14.99	0.19	14.80	30
5/8	RSPC $[8,5]^{1250}$	10^{-3}	7.2	1.8	5.4	2
5/8	RSPC $[8,5]^{1250}$	10^{-6}	10.7	1.8	8.9	7
3/4	SPC $[4,3]^{2500}$	10^{-3}	7.62	3.39	4.23	2
3/4	SPC $[4,3]^{2500}$	10^{-6}	10.91	3.39	7.52	5
7/8	SPC $[8,7]^{1250}$	10^{-3}	8.18	5.27	2.91	1
7/8	SPC $[8,7]^{1250}$	10^{-6}	11.20	5.27	5.93	3

A Construction Example – BMST-RSPC over BPSK with rate-$1/2$

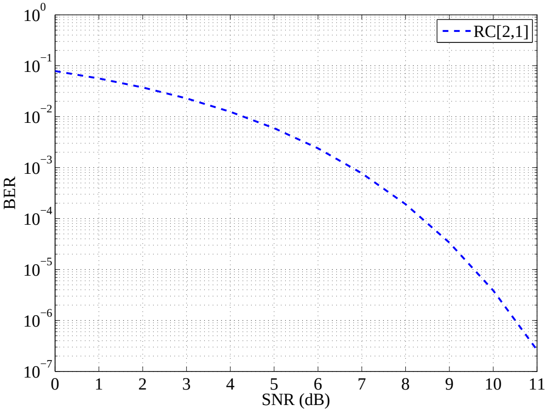

Figure: Performance of the BMST systems with the R code $[2, 1]^{5000}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

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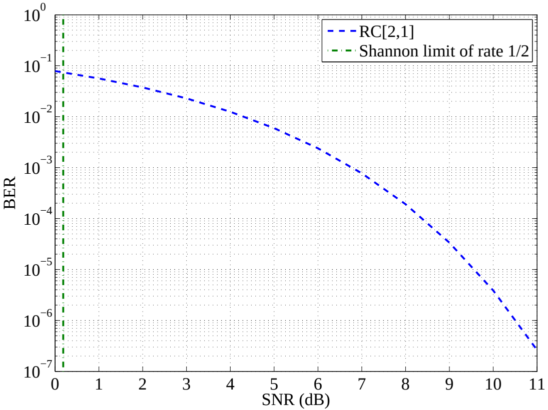

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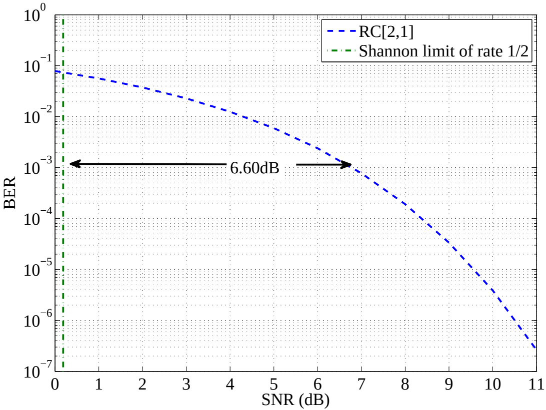

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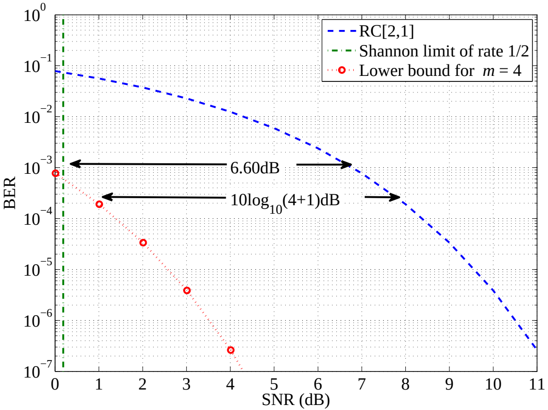

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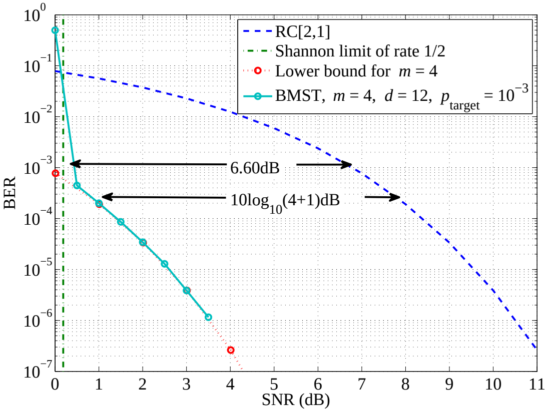

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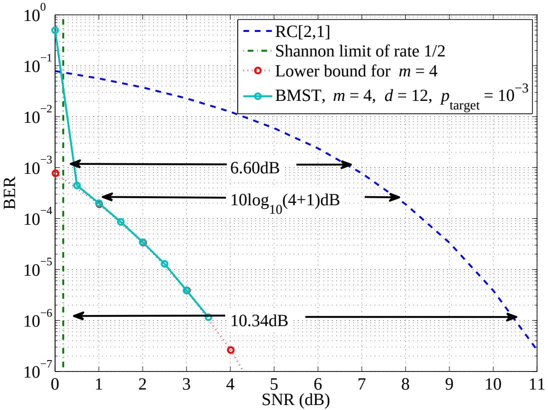

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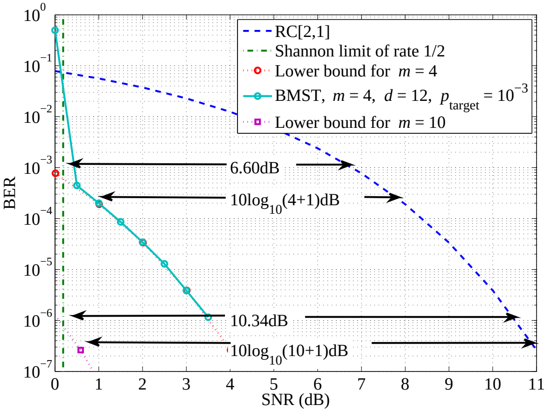

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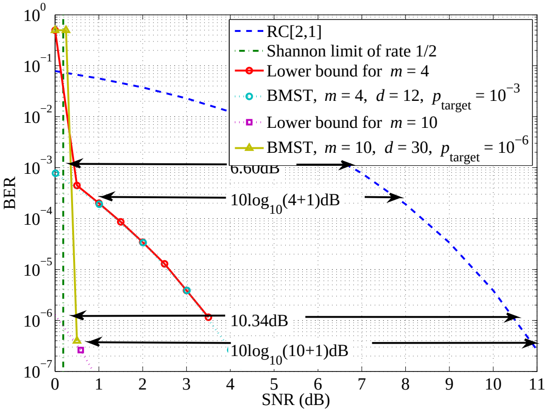

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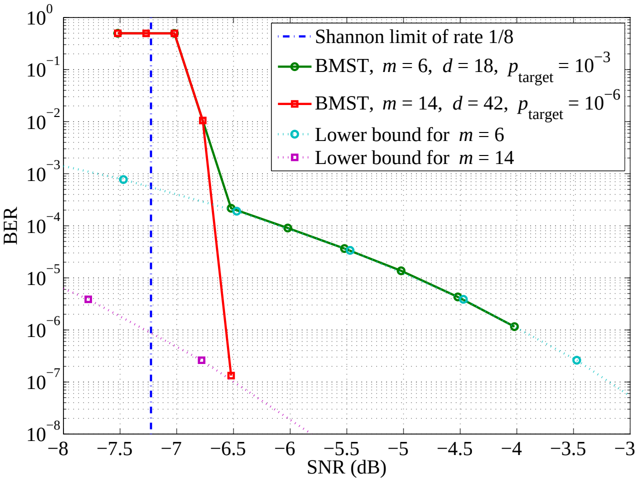


Figure: Performance of the BMST systems with the R code $[8,1]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

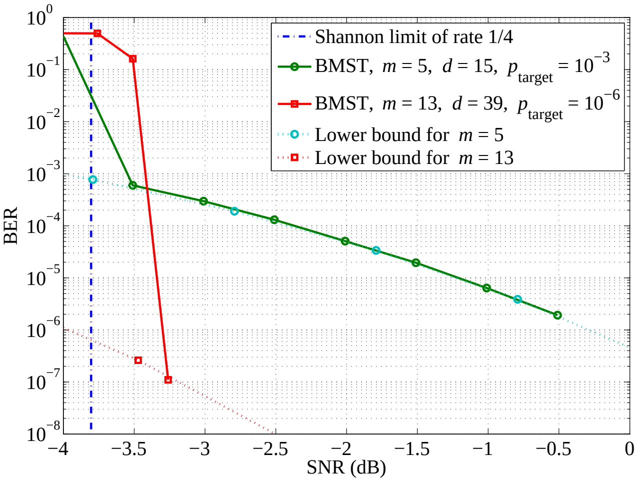


Figure: Performance of the BMST systems with the R code $[4, 1]^{2500}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

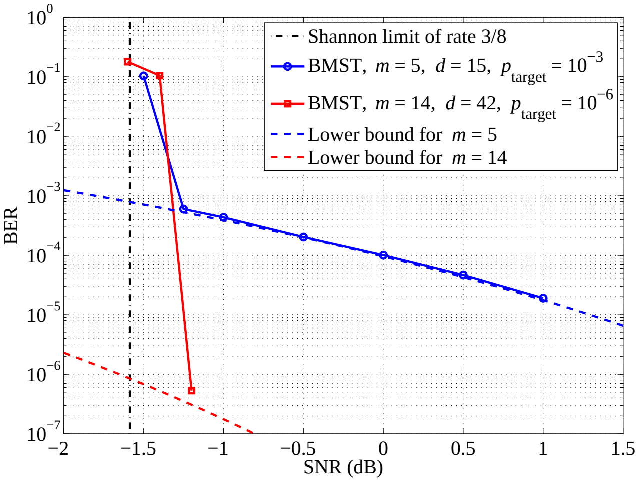


Figure: Performance of the BMST systems with the RSPC $[8,3]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

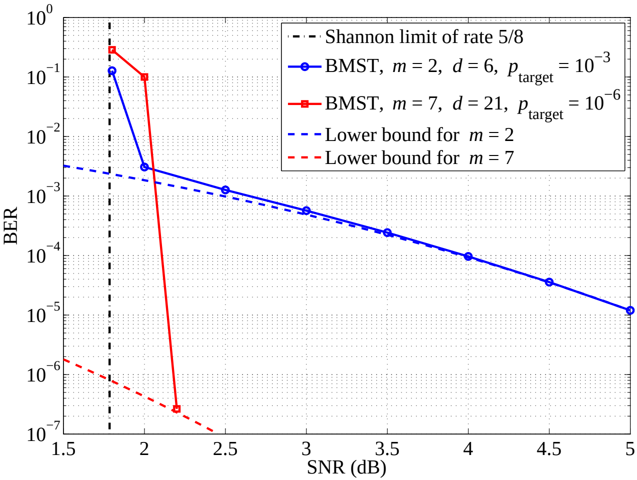


Figure: Performance of the BMST systems with the RSPC $[8,5]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

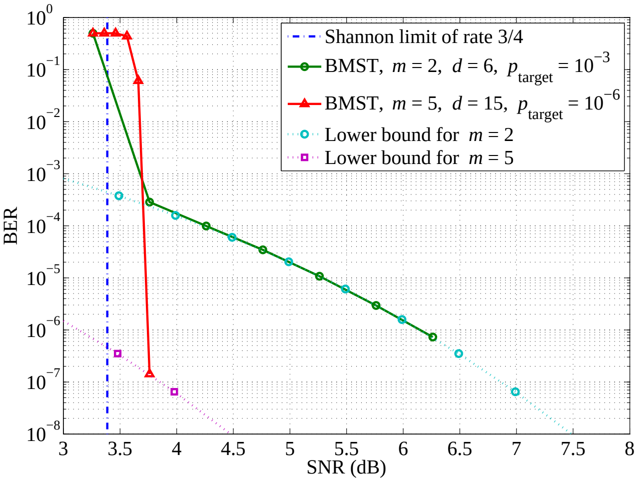


Figure: Performance of the BMST systems with the SPC $[4,3]^{2500}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

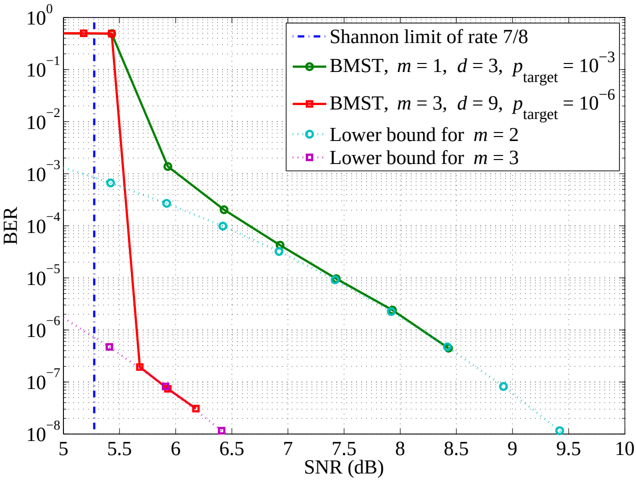


Figure: Performance of the BMST systems with the SPC $[8, 7]^{1250}$ as the basic code. The target BERs are 10^{-3} and 10^{-6}. The systems encode $L = 100000$ sub-blocks of data and decode with the SWD algorithm of a maximum iteration $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over BPSK

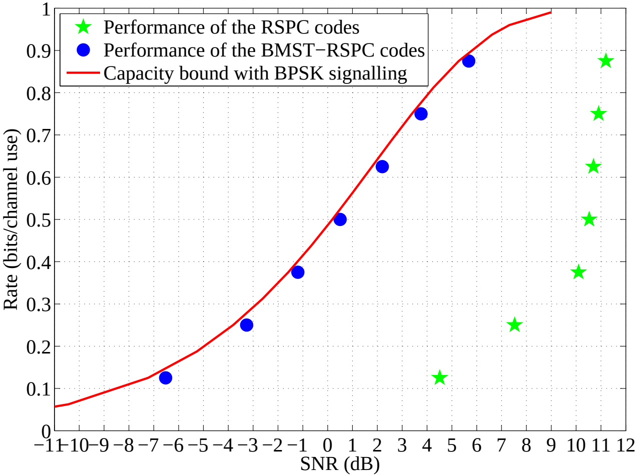


Figure: The required SNRs $(1/\sigma^2)$ for the BMST-RPSC codes to achieve the BER of 10^{-6} over the BPSK-AWGN channels.

Construction Examples – BMST-RSPC codes over BPSK

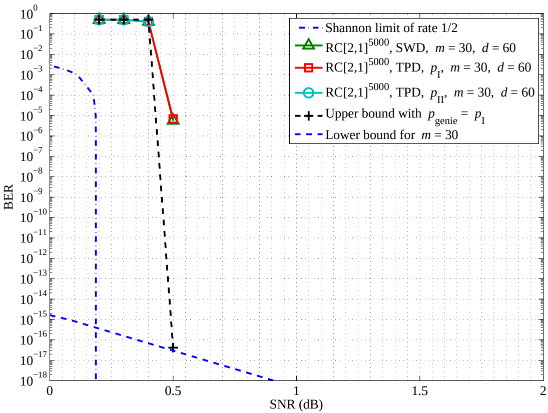


Figure: Performance of the BMST system with the R code $[2,1]^{5000}$ as the basic code. The target BER is 10^{-15}. The system encodes $L = 100000$ sub-blocks of data with the encoding memory $m = 30$ and decodes with a decoding delay $d = 60$ and a maximum iteration $I_{\max} = 18$. At the SNR of 0.5 dB, $p_{\rm I} = 7.2 \times 10^{-6}$. Hence, according to the genie-aided bound, $p_{\rm II} = 4.2 \times 10^{-17}$.

Construction Examples – BMST-RSPC codes over 8-PSK

Example

Table: The encoding memories required to approach the corresponding Shannon limits using BMST-RSPC codes with rates $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ at the target SER $p_{\text{target}} = 10^{-4}$ over the 8-PSK-AWGN channels

Rates R	p_{target}	γ_{target} (dB)	γ_{lim} (dB)	Gap $\gamma_{\text{target}} - \gamma_{\text{lim}}$ (dB)	Memory m
$1/5$	10^{-4}	10.1	−2.8	12.9	18
$2/5$	10^{-4}	13.7	1.3	12.4	16
$3/5$	10^{-4}	14.3	4.7	9.6	8
$4/5$	10^{-4}	14.8	8.1	6.7	4

Construction Examples – BMST-RSPC codes over 8-PSK

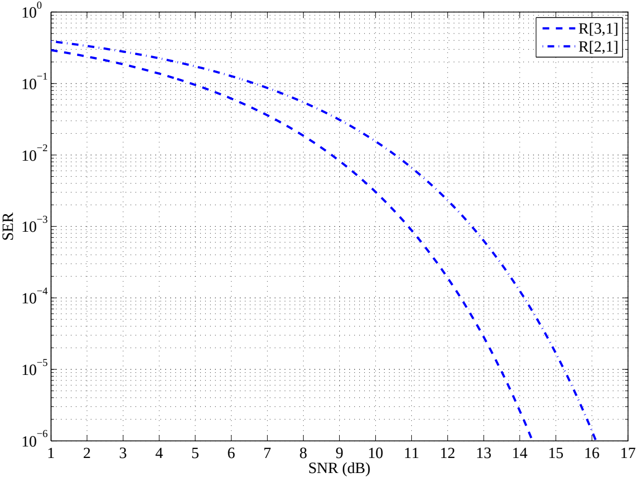
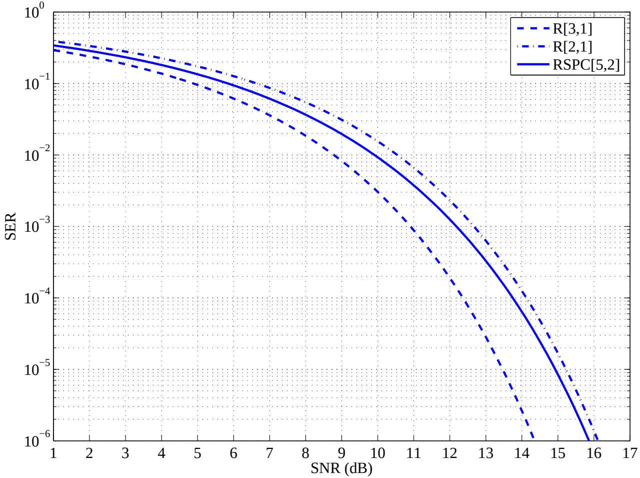


Figure: Performance of the BMST-RSPC codes using the RSPC code $[5,2]^{150}$ to achieve the target SER $p_{\text{target}} = 10^{-4}$ over the 8-PSK-AWGN channels, where the encoder terminates every $L = 1000$ sub-blocks and the maximum iteration number $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over 8-PSK



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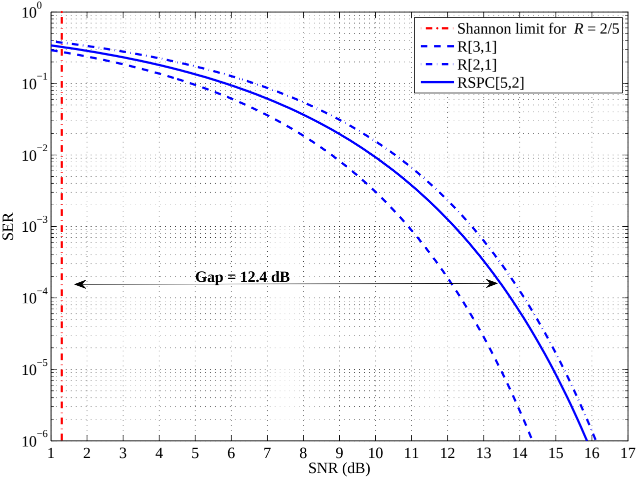

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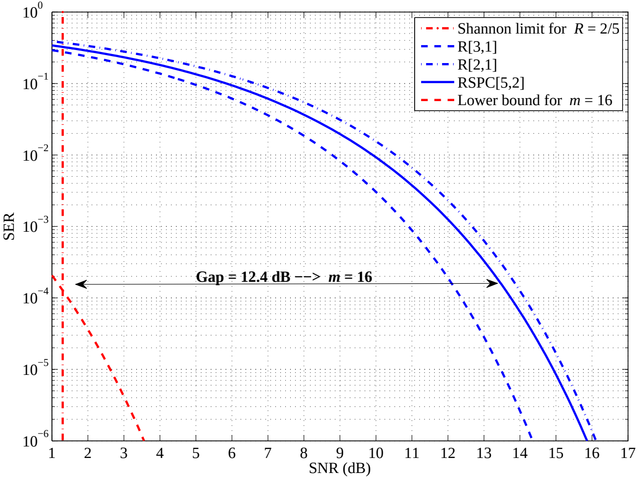

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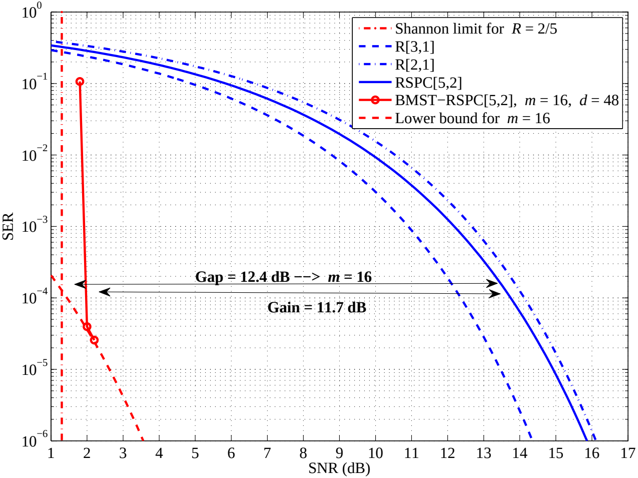

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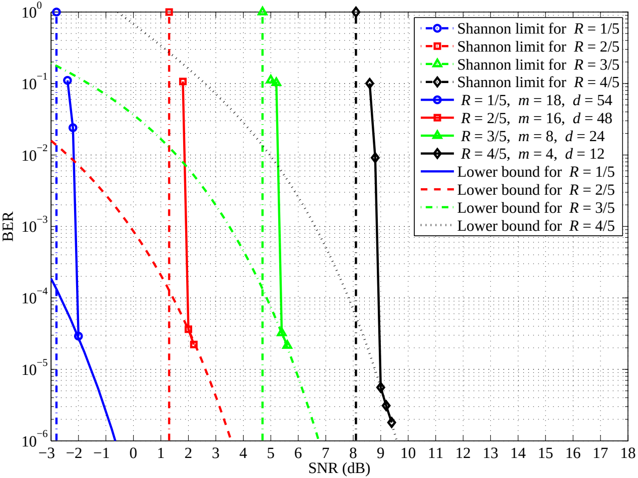

Figure: Performance of the BMST-RSPC codes using the RSPC codes $[5, K]^{150}(K = 1, 2, 3, 4)$ to achieve the target SER $p_{\text{target}} = 10^{-4}$ over the 8-PSK-AWGN channels, where the encoder terminates every $L = 1000$ sub-blocks and the maximum iteration number $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over 8-PSK

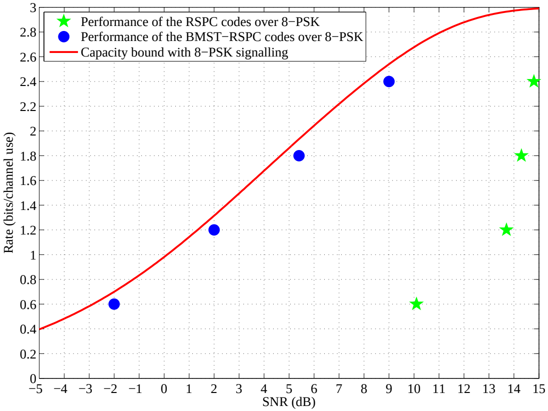

Figure: The required SNRs for the BMST-RSPC codes using the RSPC codes $[5, K]^{150}(K = 1, 2, 3, 4)$ to achieve the target SER $p_{\text{target}} = 10^{-4}$ over the 8-PSK-AWGN channels, where the encoder terminates every $L = 1000$ sub-blocks.

Construction Examples – BMST-RSPC codes over 16-QAM

Table: The encoding memory required to approach the Shannon limit using BMST-RSPC codes with rates $R = \frac{239}{255}$ at the target SER $p_{\text{target}} = 10^{-3}$ over 16-QAM-AWGN channels

Rates R	p_{target}	γ_{target} (dB)	γ_{lim} (dB)	Gap $\gamma_{\text{target}} - \gamma_{\text{lim}}$ (dB)	Memory m
$239/255$	10^{-3}	16.0	12.7	3.3	1

Construction Examples – BMST-RSPC codes over 16-QAM

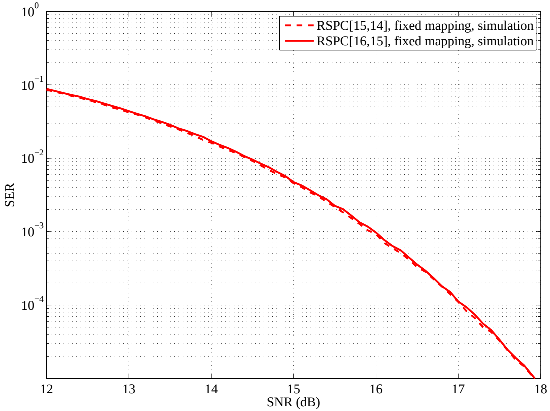

Figure: Performance of the BMST system using the RSPC code $[255, 239]^4$ to achieve the target SER $p_{\text{target}} = 10^{-3}$ over the 16-QAM-AWGN channels, where the encoder terminates every $L = 1000$ sub-blocks and the maximum iteration number $I_{\max} = 18$.

Construction Examples – BMST-RSPC codes over 16-QAM

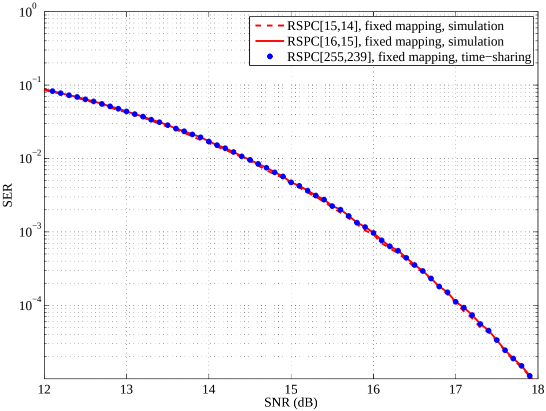

Figure: Performance of the BMST system using the RSPC code $[255, 239]^4$ to achieve the target SER $p_{\text{target}} = 10^{-3}$ over the 16-QAM-AWGN channels, where the encoder terminates every $L = 1000$ sub-blocks and the maximum iteration number $I_{\text{max}} = 18$.

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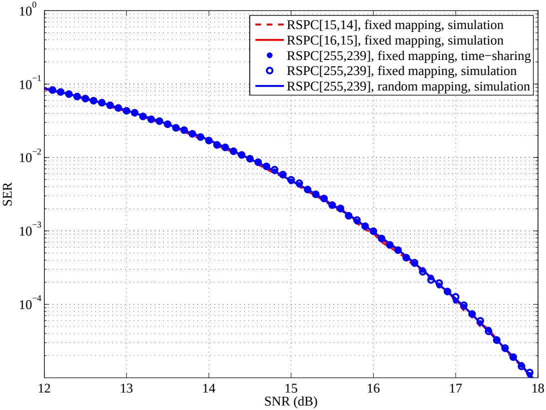

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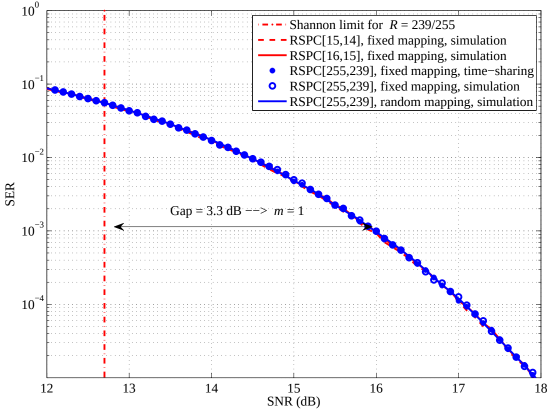

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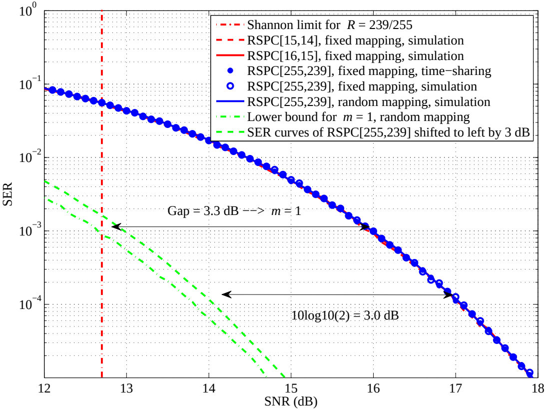

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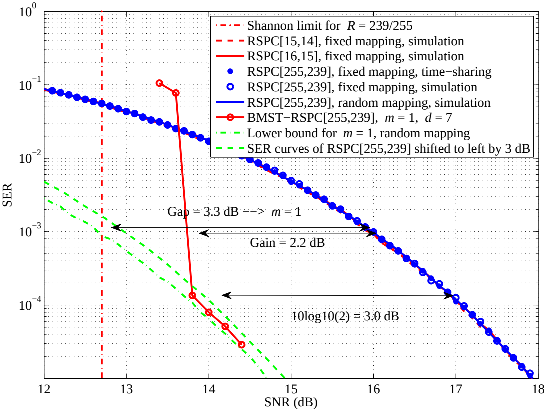

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Outline

Conclusions

Conclusions

- We have presented a simple (also deterministic) procedure to construct codes
  - for any rational code rate;
  - over any alphabet;
  - performing well at any given target error rate;
  - having linear complexity with the code length.
- Generalization to other *ergodic* channels is possible.

Related Works

- Xiao Ma, Chulong Liang, Kechao Huang, and Qiutao Zhuang, "Obtaining extra coding gain for short codes by block Markov superposition transmission," in Proceeding IEEE International Symposium on Information Theory, Istanbul, Turkey, July 2013, pp. 2054-2058.

- Xiao Ma, Chulong Liang, Kechao Huang, and Qiutao Zhuang, "Block Markov Superposition Transmission Construction of Big Convolutional Codes from Short Codes," *IEEE Trans. Inf. Theory*, under 2nd round review, July 2014.

- Chulong Liang, Xiao Ma, Qiutao Zhuang, and Baoming Bai, "Spatial coupling of generator matrices: A general approach to design good codes at a target BER," *IEEE Transactions on Communications*, October 2014, [Online]. Available: http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6940240.

- Chulong Liang, Xiao Ma, Qiutao Zhuang, and Baoming Bai, "A general procedure to design good codes at a target BER," in Proceeding *the 8th International Symposium on Turbo Codes & Iterative Information Processing*, Bremen, Germany, August 2014.

- Qiutao Zhuang, Xiao Ma, and Aleksander Kavčić, "Bounds on the ML decoding error probability of RS-Coded modulation over AWGN channels," [Online]. Available: http://arxiv.org/abs/1401.5305.

- Qiutao Zhuang, Jia Liu, and Xiao Ma, "Upper bounds on the ML decoding error probability of general codes over AWGN channels," [Online]. Available: http://arxiv.org/abs/1308.3303.

- Jingnan Hu, Chulong Liang, Xiao Ma, and Baoming Bai, "A new class of multiple-rate codes based on block Markov superposition transmission," in *Proceeding International Workshop on High Mobility*

Related Works

Chulong Liang, Jingnan Hu, Xiao Ma, and Baoming Bai, "A new class of multiple-rate codes based on block Markov superposition transmission," submitted to *IEEE Transactions on Signal Processing*, under 2nd round review, October 2014, [Online]. Available: http://arxiv.org/abs/1308.4809.

Xiying Liu, Chulong Liang, and Xiao Ma, "Block Markov superposition transmission of convolutional codes with MSK signaling," *IET Communications*, accepted, August 2014.

Zhihua Yang, Chulong Liang, Xiaopei Xu, and Xiao Ma, "Block Markov superposition transmission with spatial modulation," *IEEE Wireless Communications Letters*, accepted, August 2014.

Chulong Liang, Kechao Huang, Xiao Ma, and Baoming Bai, "Block Markov superposition transmission with bit-interleaved coded modulation," *IEEE Communications Letters*, vol. 18, no. 3, pp. 397-400, March 2014.

Thank You for Your Attention!

Acknowledgements

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